MULTIPLE-LEVEL LOGIC OPTIMIZATION

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Outline

- Representations.
- Taxonomy of optimization methods:
  - Goals: area/delay.
  - Algorithms: algebraic/Boolean.
  - Rule-based methods.
- Examples of transformations.
- Boolean and algebraic models.

Motivation

- Multiple-level networks:
  - Semi-custom libraries.
  - Gates versus macros (PLAs):
    - More flexibility.
    - Better performance.
- Applicable to a variety of designs.

Circuit modeling

- Logic network:
  - Interconnection of logic functions.
  - Hybrid structural/behavioral model.
- Bound (mapped) networks:
  - Interconnection of logic gates.
  - Structural model.

Example of bound network

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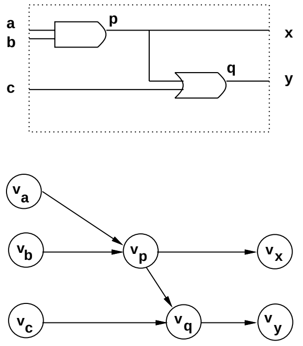

Example of network

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$$
\begin{aligned}
p &= ce + de \\
q &= a + b \\
r &= p + a' \\
s &= r + b' \\
t &= ac + ad + bc + bd + e \\
u &= q'c + qc' + qc \\
v &= a'd + bd + c'd + ae' \\
w &= v \\
x &= s \\
y &= t \\
z &= u
\end{aligned}
$$

Example of network

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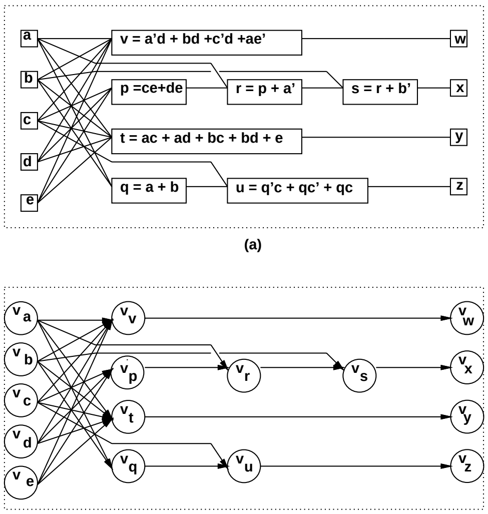

(a)

(b)

Example circuit terminal behavior

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$$
\mathbf{f} = \begin{bmatrix} a'd + bd + c'd + ae' \\ a' + b' + c + d \\ ac + ad + bc + bd + e \\ a + b + c \end{bmatrix}
$$

Network optimization

- Minimize area (power) estimate:
  - subject to delay constraints.
- Minimize maximum delay:
  - subject to area (power) constraints.
- Minimize power consumption.
  - subject to delay constraints.
- Maximize testability.

Estimation

- Area:
  - Number of literals.
  - Number of functions/gates.
- Delay:
  - Number of stages.
  - Refined gate delay models.
  - Sensitizable paths.

Problem analysis

- Multiple-level optimization is hard.
- Exact methods:
  - Exponential complexity.
  - Impractical.
- Approximate methods:
  - Heuristic algorithms.
  - Rule-based methods.

Strategies for optimization

- Improve circuit step by step.
  - Circuit *transformations*.
- Preserve network behavior.
- Methods differ in:
  - *Types* of transformations.
  - *Selection* and *order* of transformations.

Example elimination

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- Eliminate one function from the network.
- Perform variable substitution.
- Example:
  - $s = r + b'; \quad r = p + a'$
  - $\Rightarrow s = p + a' + b'$.

Example elimination

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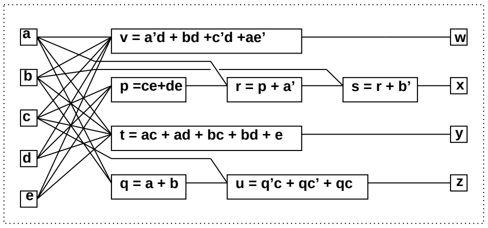

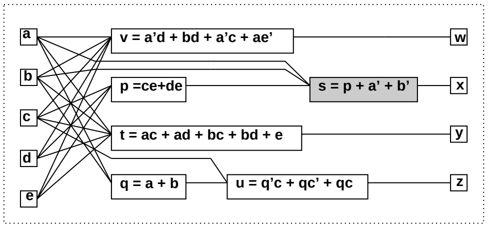

Example decomposition

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- Break one function into smaller ones.
- Introduce new vertices in the network.
- Example:
  - $v = a'd + bd + c'd + ae'$.
  - $\Rightarrow j = a' + b + c'; v = jd + ae'$

Example decomposition

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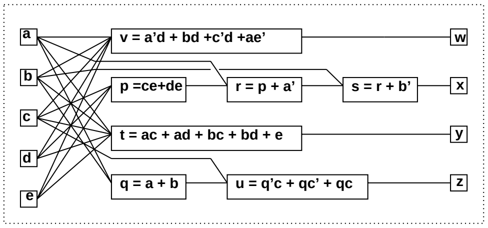
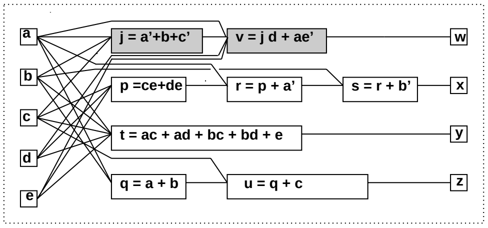
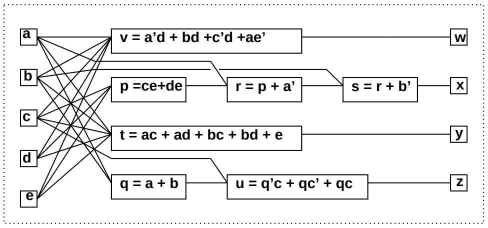


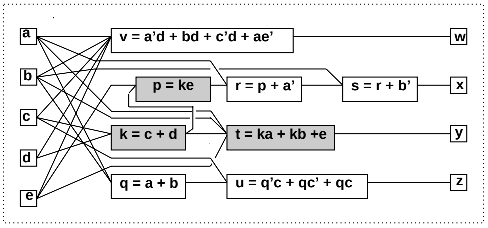


Example extraction

- Find a common sub-expression of two (or more) expressions.
- Extract sub-expression as new function.
- Introduce new vertex in the network.
- Example:
  - $p = ce + de; \quad t = ac + ad + bc + bd + e;$
  - $p = (c + d)e; \quad t = (c + d)(a + b) + e;$
  - $\Rightarrow k = c + d; \quad p = ke; \quad t = ka + kb + e;$

Example extraction

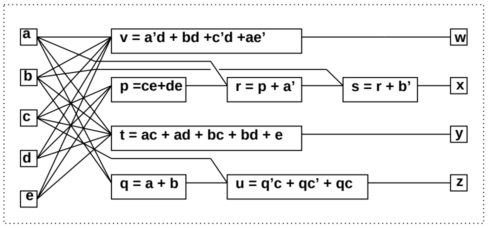

a
b
c
d
e
v = a'd + bd +c'd +ae'
p =ce+de
r = p + a'
s = r + b'
t = ac + ad + bc + bd + e
q = a + b
u = q'c + qc' + qc
w
x
y
z


a
b
c
d
e
v = a'd + bd + c'd + ae'
p = ke
r = p + a'
s = r + b'
k = c + d
t = ka + kb +e
q = a + b
u = q'c + qc' + qc
w
x
y
z

Example simplification

- Simplify a local function.
- Example:
  - $u = q'c + qc' + qc;$
  - $\Rightarrow u = q + c;$

Example simplification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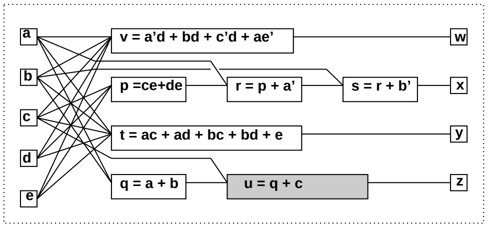
a
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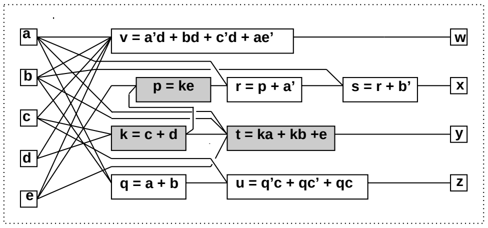
p =ce+de
r = p + a'
s = r + b'
t = ac + ad + bc + bd + e
q = a + b
u = q + c
w
x
y
z

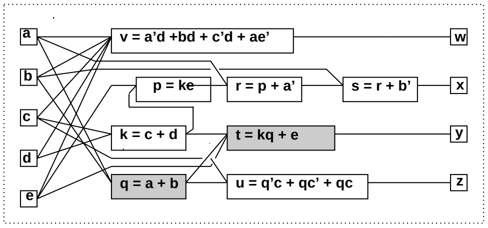
Example substitution

- Simplify a local function by using an additional input that was not previously in its support set.

- Example:
  - $t = ka + kb + e$.
  - $\Rightarrow t = kq + e$
  - Because $q = a + b$.

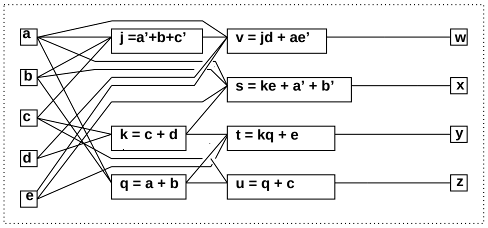
Example substitution





Example sequence of transformations

$$\begin{array}{rcl}
j & = & a' + b + c' \\
k & = & c + d \\
q & = & a + b \\
s & = & ke + a' + b' \\
t & = & kq + e \\
u & = & q + c \\
v & = & jd + ae'
\end{array}$$



Optimization approaches

- *Algorithmic* approach:
  - Define an algorithm for each transformation type.
  - Algorithm is an *operator* on the network.

- *Rule-based* approach:
  - Rule-data base:
    - ∗ Set of pattern pairs.
  - Pattern replacement driven by rules.

Algorithmic approach

- Each operator has well-defined properties:
  - Heuristic methods still used.
  - Weak optimality properties.
- Sequence of operators:
  - Defined by *scripts*.
  - Based on experience.

Example elimination algorithm

- Set a threshold k (usually 0).
- Examine all expressions.
- Eliminate expressions if the increase in literals does not exceed the threshold.

Example elimination algorithm

```
$ELIMINATE( G_n(V,E) , k)\{$
     **repeat** $\{$
          $v_x$ = selected vertex with value $< k$;
          **if** $(v_x = \emptyset)$ **return**;
          replace $x$ by $f_x$ in the network;
     $\}$
$\}$
```

Example MIS/SIS rugged script

- sweep; eliminate -1
- simplify -m nocomp
- eliminate -1
- sweep; eliminate 5
- simplify -m nocomp
- resub -a
- fx
- resub -a; sweep
- eliminate -1; sweep
- full-simplify -m nocomp

Boolean and algebraic methods

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- Boolean methods:
  - Exploit properties of logic functions.
  - Use *don't care* conditions.
  - Complex at times.
- Algebraic methods:
  - View functions as *polynomials.*
  - Exploit properties of polynomial algebra.
  - Simpler, faster but weaker.

Example

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- Boolean substitution:
  - $h = a + bcd + e; \; q = a + cd$
  - $\Rightarrow h = a + bq + e$
  - Because $a + bq + e = a + b(a + cd) + e =$ $= a + bcd + e.$
- Algebraic substitution:
  - $t = ka + kb + e.$
  - $\Rightarrow t = kq + e$
  - Because $q = a + b.$

Summary

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- Multilevel logic synthesis is performed by step-wise transformations.
- Algorithms are based on both the Boolean and the algebraic models.
- Rule-based systems.