SCHEDULING II

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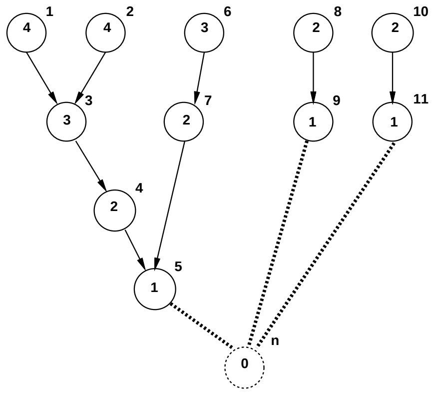
Scheduling under resource constraints

- Simplified models:
  - Hu's algorithm.
- Heuristic algorithms:
  - List scheduling.
  - Force-directed scheduling.

Hu's algorithm

- Assumptions:
  - Graph is a forest.
  - All operations have unit delay.
  - All operations have the same type.
- Algorithm:
  - Label vertices with distance from sink.
  - Greedy strategy.
  - Exact solution.

Example



- Assumptions:
  - One resource type only.
  - All operations have unit delay.

Algorithm
Hu's schedule with $\overline{a}$ resources

- Set step $l = 1$.
- Repeat until all ops are scheduled:
  - Select $s \leq \overline{a}$ resources with:
    * All predecessors scheduled.
    * Maximal labels.
  - Schedule the s operations at step l.
  - Increment step $l = l + 1$.

Example

- Minimum latency with $a = 3$ resources.
- Step 1: Select $\{v_1, v_2, v_6\}$.
- Step 2: Select $\{v_3, v_7, v_8\}$.
- Step 3: Select $\{v_4, v_9, v_{10}\}$.
- Step 4: Select $\{v_5, v_{11}\}$.

Exactness of Hu's algorithm

- **Theorem1:**
  - Given a dag with ops of the same type.
  - $\overline{a} = \max_{\gamma} \lceil \frac{\sum_{j=1}^{\gamma} p(\alpha + 1 - j)}{\gamma + \lambda - \alpha} \rceil$
  - $\overline{a}$ is a lower bound on the number of resources to complete a schedule with latency λ.
  - γ is a positive integer.
- **Theorem2:**
  - Hu's algorithm applied to a tree with unit-cycle resources achieves latency λ with $\overline{a}$ resources.
- **Corollary:**
  - Since $\overline{a}$ is a lower bound on the number of resources for achieving λ, then λ is minimum.

List scheduling algorithms

- Heuristic method for:
  - Min *latency* subject to *resource bound.*
  - Min *resource* subject to *latency bound.*
- Greedy strategy (like Hu's).
- General graphs (unlike Hu's).
- Priority list heuristics.
  - Longest path to sink.
  - Longest path to timing constraint.

List scheduling algorithm for minimum latency

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LIST_L($G(V, E), \mathbf{a}$) {
  $l = 1$;
  repeat {
    for each resource type $k = 1, 2, \ldots n_{res}$ {
      Determine candidate operations $U_{l,k}$;
      Determine unfinished operations $T_{l,k}$;
      Select $S_k \subseteq U_{l,k}$ vertices, s.t. $|S_k| + |T_{l,k}| \leq a_k$;
      Schedule the S_k operations at step l;
      }
    $l = l + 1$;
  }
  until (v_n is scheduled) ;
  return ($\mathbf{t}$);
}

Example

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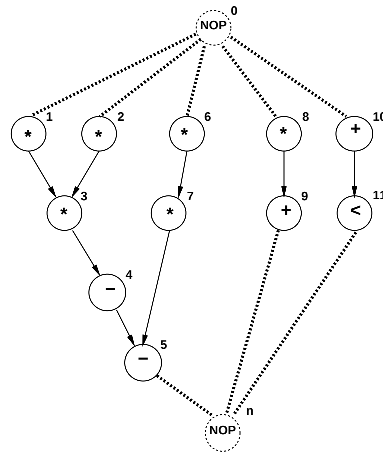

- Assumptions:
  - $a_1 = 3$ multipliers with delay 2.
  - $a_2 = 1$ ALUs with delay 1.

Example

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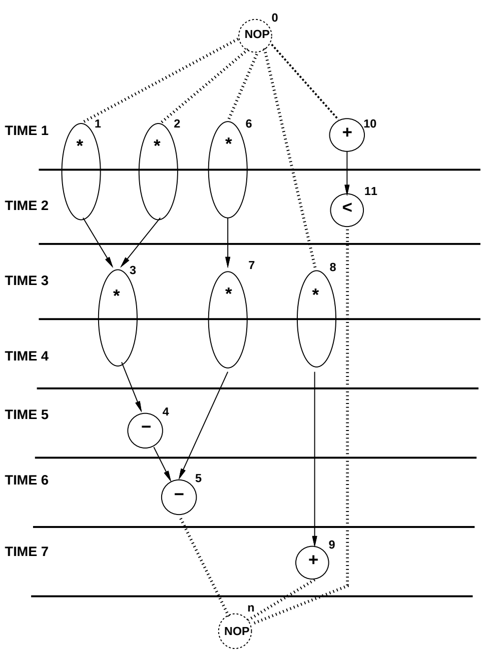

List scheduling algorithm for minimum resource usage

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LIST_R($G(V, E), \overline{\lambda}$) {
  $\mathbf{a} = \mathbf{1}$;
  Compute the latest possible start times $\mathbf{t}^L$
  by ALAP ($G(V, E), \overline{\lambda}$);
  if ($t_0^L < 0$)
    return ($\emptyset$);
  $l = 1$;
  repeat {
    for each resource type $k = 1, 2, \ldots n_{res}$ {
      Determine candidate operations U_{lk};
      Compute the slacks $\{s_i = t_i^L - l \ \forall v_i \in U_{lk}\}$;
      Schedule the candidate operations
      with zero slack and update $\mathbf{a}$;
      Schedule the candidate operations
      that do not require additional resources;
    }
    $l = l + 1$;
  }
  until (v_n is scheduled) ;
  return ($\mathbf{t}, \mathbf{a}$);
}

Example

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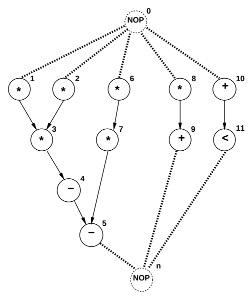

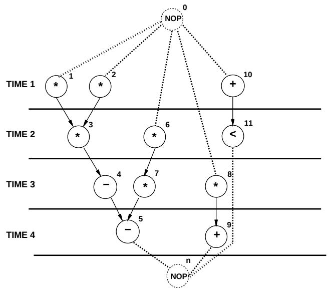

Force-directed scheduling

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- Heuristic scheduling methods [Paulin]:
  - Min *latency* subject to *resource bound.*
    - Variation of list scheduling: FDLS.
  - Min *resource* subject to *latency bound.*
    - Schedule one operation at a time.
- Rationale:
  - Reward *uniform distribution* of operations across schedule steps.

Force-directed scheduling definitions

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- Operation *interval*: mobility plus one (μ_i+1).
  - Computed by ASAP and ALAP scheduling $[t_i^S, t_i^L]$.
- *Operation probability* $p_i(l)$:
  - Probability of executing in a given step.
  - $1/(\mu_i + 1)$ inside interval; 0 elsewhere.
- *Operation-type distribution* $q_k(l)$:
  - Sum of the op. prob. for each type.

Example

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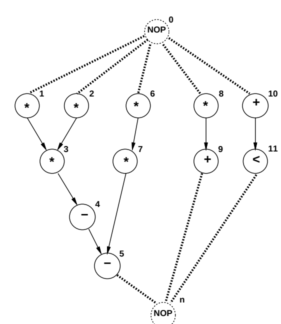

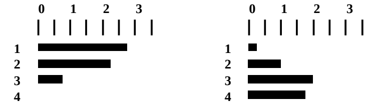

- Distribution graphs for multiplier and ALU.

Force

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- Used as *priority* function.
- Force is related to concurrency.
  - Sort operations for least force.
- Mechanical analogy:
  - Force = *constant* × *displacement*.
    - *constant* = operation-type distribution.
    - *displacement* = change in probability.

Forces related to the assignment of an operation to a control step

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- Self-force:
  - Sum of forces to other steps.
  - Self-force for operation v_i in step l:
    - $\sum_{m=t_i^S}^{t_i^L} q_k(m)(\delta_{lm} - p_i(m))$
- Successor-force:
  - Related to the successors.
  - Delaying an operation implies delaying its successors.

Example: operation v_6

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- It can be scheduled in the first two steps.
  - $p(1) = 0.5; p(2) = 0.5; p(3) = 0; p(4) = 0$.
- Distribution: $q(1) = 2.8; q(2) = 2.3$.
- Assign v_6 to step 1:
  - variation in probability $1 - 0.5 = 0.5$ for step 1.
  - variation in probability $0 - 0.5 = -0.5$ for step 2.
- Self-force: $2.8 * 0.5 - 2.3 * 0.5 = +0.25$

Example: operation v_6

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- Assign v_6 to step 2:
  - variation in probability $0 - 0.5 = -0.5$ for step 1.
  - variation in probability $1 - 0.5 = 0.5$ for step 2.
- Self-force: $-2.8 * 0.5 + 2.3 * 0.5 = -0.25$

Example: operation v_6

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- Successor-force:
  - Operation v_7 assigned to step 3.
  - 2.3 (0-0.5) + 0.8 (1 -0.5) = -.75
- Total-force = -1.
- Conclusion:
  - Least force is for step 2.
  - Assigning v_6 to step 2 reduces concurrency.

Force-directed scheduling algorithm for minimum resources

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```
FDS( G(V,E), λ̄ ) {
   repeat {
      Compute the time-frames;
      Compute the operation and type probabilities;
      Compute the self-forces, p/s-forces and total forces;
      Schedule the op. with least force, update time-frame;
   } until (all operations are scheduled)
   return (t);
}
```

Scheduling with chaining

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- Consider propagation delays of resources not in terms of cycles.
- Use scheduling to *chain* multiple operations in the same control step.
- Useful technique to explore effect of *cycle-time* on area/latency trade-off.
- Algorithms:
  - ILP, ALAP/ASAP, List scheduling.

Example

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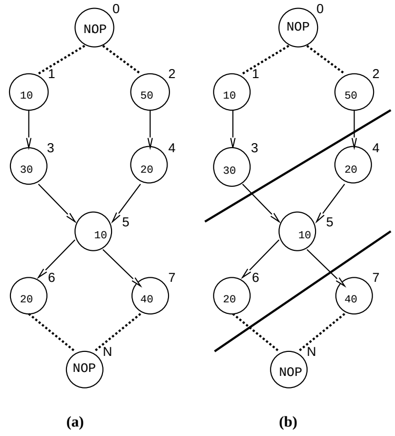

- Cycle-time: 60.

Summary

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- Scheduling determines *area/latency* trade-off.
- Intractable problem in general:
  - Heuristic algorithms.
  - ILP formulation (small-case problems).
- Chaining:
  - Incorporate *cycle-time* considerations.