TIMING ISSUES IN MULTI-LEVEL LOGIC OPTIMIZATION

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Outline

- Timing verification.
  - Delay modeling.
  - Critical paths.
  - The false path problem.
- Algorithms for timing optimization.

Timing verification and optimization

- Verification:
  - Check that a circuit runs at speed:
    - Satisfies I/O delay constraints.
    - Satisfies cycle-time constraints.
- Optimization:
  - *Minimum area*
    - subject to *delay* constraints.
  - *Minimum delay*
    - (subject to *area* constraints).

Delay modeling

- Gate delay modeling:
  - Straightforward for bound networks.
  - Approximations for unbound networks.
- Network delay modeling:
  - Compute signal propagation:
    - Topological methods.
    - Logic/topological methods.

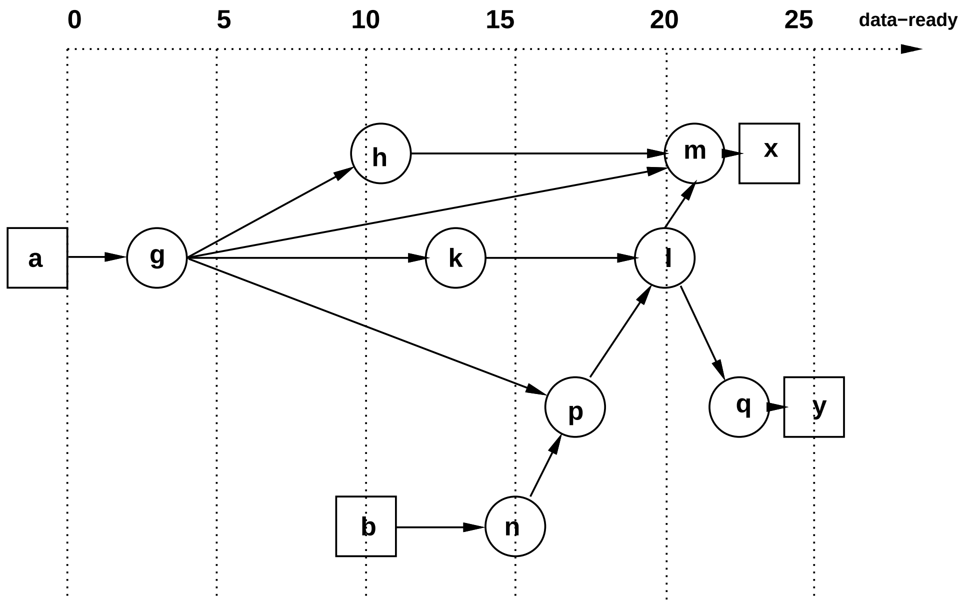
Gate delay modeling unbound networks

- Virtual gates:
  - Logic expressions.
- Stage delay model:
  - Unit delay per vertex.
- Refined models:
  - Depending on fanout.

Network delay modeling

- For each vertex v_i.
- *Propagation delay* d_i.
  - I/O propagation delays are usually zero.
- *Data-ready time* t_i.
  - Input data-ready times denote when inputs are available.
  - Computed elsewhere by *forward traversal*:
  - $t_i = d_i + \max_{j|(v_j,v_i)\in E} t_j$

Example



- Propagation delays:
  - $d_g = 3; d_h = 8; d_m = 1; d_k = 10; d_l = 3;$
  - $d_n = 5; d_p = 2; d_q = 2; d_x = 2; d_y = 3;$

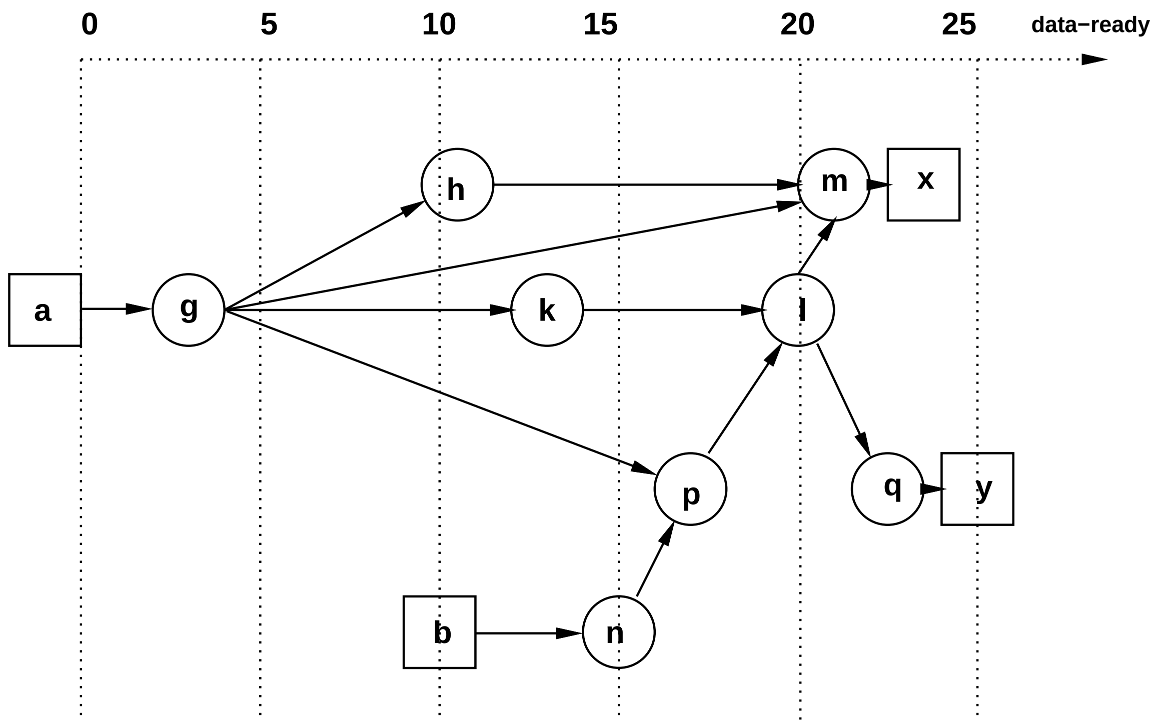
Network delay modeling

- For each vertex v_i.

- *Required data-ready time* $\bar{t}_x$.
  - Specified at the primary outputs.
  - Computed elsewhere by *backward traversal*:
  - $\bar{t}_i \; = \; \min_{j|(v_i,v_j)\in E} \; \bar{t}_j - d_j$

- *Slack* s_i.
  - Difference between required and actual data-ready times $s_i = \bar{t}_i - t_i$.

Example



- Required data-ready times:
  - $\bar{t}_x = 25$ and $\bar{t}_y = 25$.

Example

- $s_x = 2; s_y = 0$
- $\bar{t}_m = 25 - 2 = 23; s_m = 23 - 21 = 2;$
- $\bar{t}_q = 25 - 3 = 22; s_q = 22 - 22 = 0;$
- $\bar{t}_l = \min\{23 - 1; 22 - 2\} = 20; s_l = 20 - 20 = 0;$
- $\bar{t}_h = 23 - 1 = 22; s_h = 22 - 11 = 11;$
- $\bar{t}_k = 20 - 3 = 17; s_k = 17 - 13 = 4;$
- $\bar{t}_p = 20 - 3 = 17; s_p = 17 - 17 = 0;$
- $\bar{t}_n = 17 - 2 = 15; s_n = 15 - 15 = 0;$
- $\bar{t}_b = 15 - 5 = 10; s_b = 10 - 10 = 0;$
- $\bar{t}_g = \min\{22 - 11; 17 - 10; 17 - 2\} = 7; s_g = 7 - 3 = 4;$
- $\bar{t}_a = 7 - 3 = 4; s_b = 4 - 0 = 4.$

Topological critical path

- Assume topologic computation of:
  - Data-ready by forward traversal.
  - Required data-ready by backward traversal.

- *Topological critical path*:
  - Input/output path with zero slacks.
  - Any increase in the vertex propagation delay affects the output data-ready time.

- A topological critical path may be *false*.
  - No event can propagate along that path.

Example

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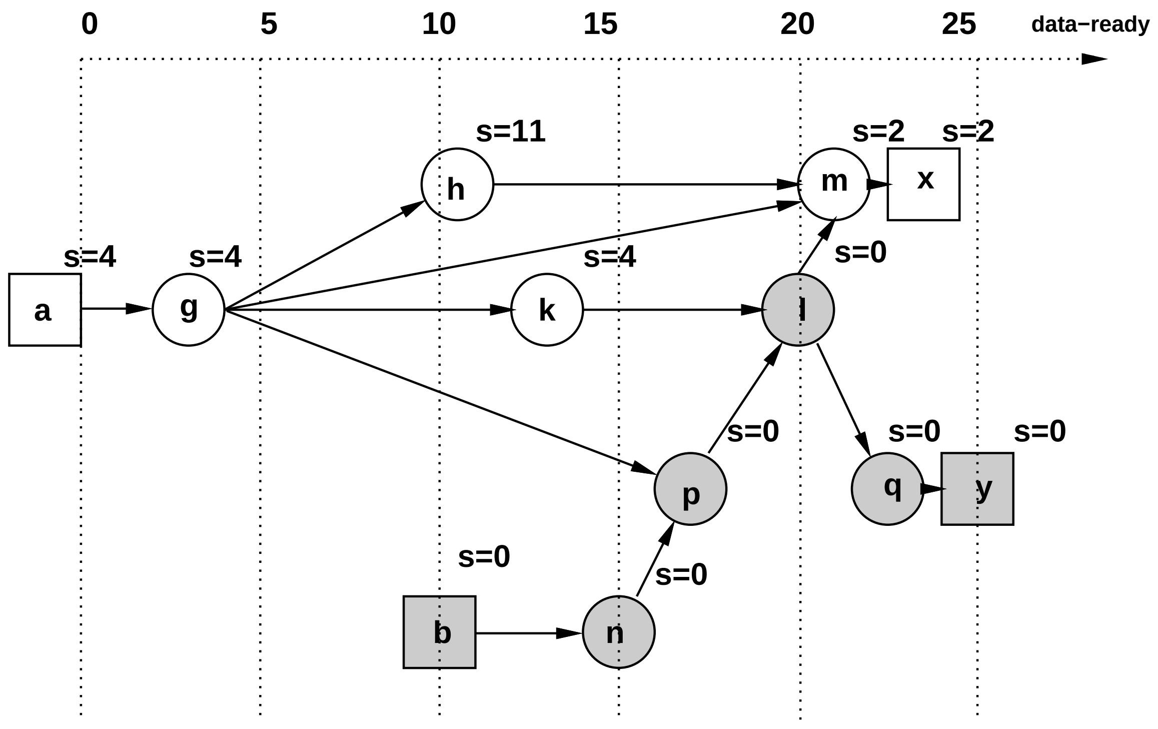
0
5
10
15
20
25
data−ready
s=11
s=2
s=2
h
m
x
s=4
s=4
s=4
s=0
a
g
k
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s=0
s=0
s=0
p
q
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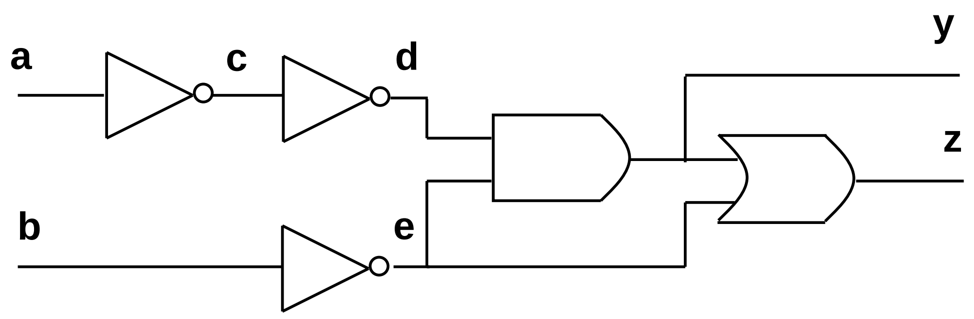
s=0
s=0
b
n

Example



- All gates have unit delay.
- All inputs ready at time 0.
- Longest topological path: $(v_a, v_c, v_d, v_y, v_z)$.
  - Path delay: 4 units.
- Critical true path: (v_a, v_c, v_d, v_y).
  - Path delay: 3 units.

Sensitizable paths

- A path in a logic network is *sensitizable* if an event can propagate from its tail to its head.
- A *critical path* is a sensitizable path of maximum weight.
- Only sensitizable paths should be considered.
- Non-sensitizable paths are *false* and can be discarded.

Sensitizable paths

- *Path*:
  - Ordered set of vertices.
- *Inputs* to a vertex:
  - Direct predecessors.
- *Side-inputs* of a vertex:
  - Inputs not on the path.

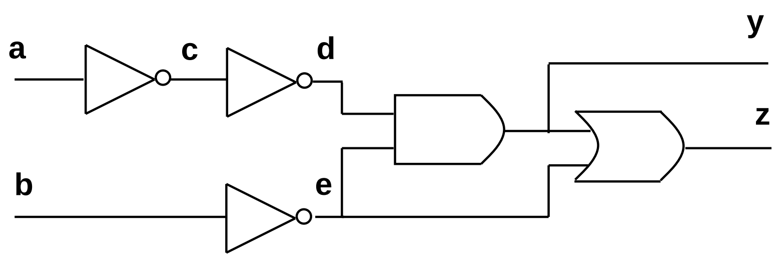
Dynamic sensitization condition

- Path: $P = (v_{x_0}, v_{x_1}, \ldots, v_{x_m})$.

- An event propagates along P if
  - $\partial f_{x_i} / \partial x_{i-1} \; = 1 \; \forall i = 1, 2, \ldots, m$.

- Remark:
  - Boolean differences are function of the side-inputs and values on the side-inputs may change.
  - Boolean differences must be true *at the time that the event propagates.*

Example



- Path: $(v_a, v_c, v_d, v_y, v_z)$
  - $\partial f_y / \partial d = e = 1$ at time 2.
  - $\partial f_z / \partial y = e' = 1$ at time 3.
- Not dynamically sensitizable because e settles at time 1.

Static sensitization

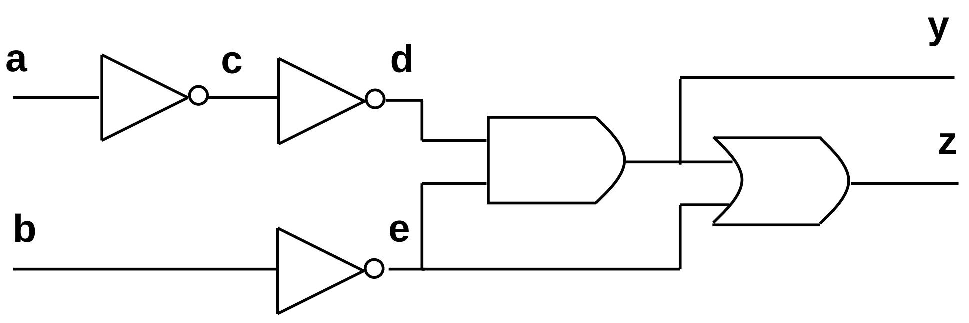
- Simpler, weaker model.

- We neglect the requirement on *when* the Boolean differences must be true to propagate an event.

- There is an assignment of primary inputs **c** such that $\partial f_{x_i}(\mathbf{c})/\partial x_{i-1} = 1 \ \forall i = 1, 2, \ldots, m$.

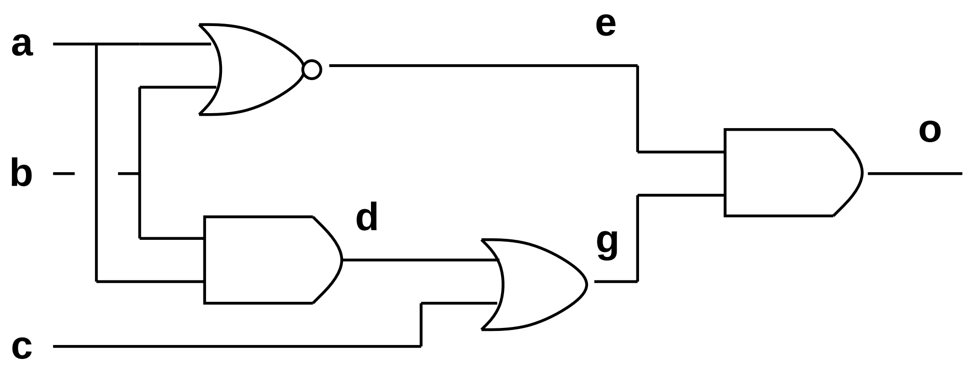
- May lead to *underestimate* delays.

Example



- Not statically sensitizable.

Example



- All gates have unit propagation delay.

Example

- Topological critical paths:
  - $\{(v_a, v_d, v_g, v_o); (v_b, v_d, v_g, v_o)\}$
  - Path delay: 3.
  - Not statically sensitizable.
- Other path:
  - (v_a, v_e, v_o)
  - Path delay: 2.
- Assume:
  - $c = 0$ and a, b dropping from 1 to 0.
  - Event propagates to output !!!

Modes for delay computation

- *Transition mode*:
  - Variables assumed to hold previous values.
    - * Model circuit node capacitances.
  - Need *two* input vectors to test.

- *Floating mode*:
  - Circuit is assumed to be memoryless.
  - Need *only one* test vector.
  - Variables have unknown value until set by input test vector.

Modes for delay computation

- *Floating mode* delay computation is simpler than *transition mode* computation.

- *Floating mode* is a pessimistic approach.

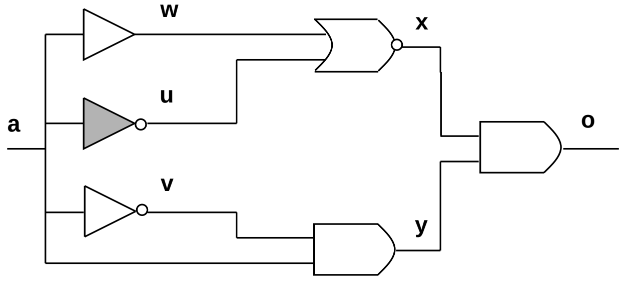
- *Floating mode* is more robust:

  – *Transition mode* may not have the *monotone speed-up* property.

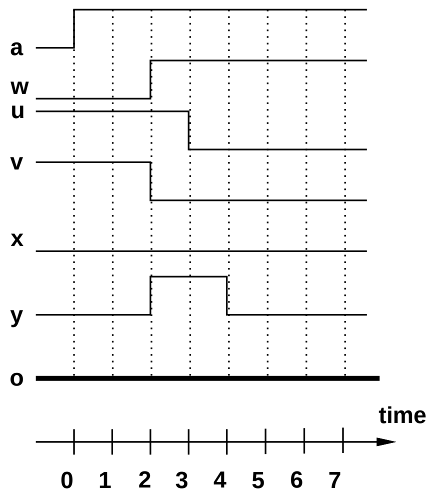
Monotone speed-up property

- Propagation delays are upper bounds.
  - What happens if gates are *faster* than expected?
- We must insure that speeding-up a gate does not slow-down the circuit.
  - Topological critical paths are robust.
  - What about dynamically sensitizable paths in transition mode?

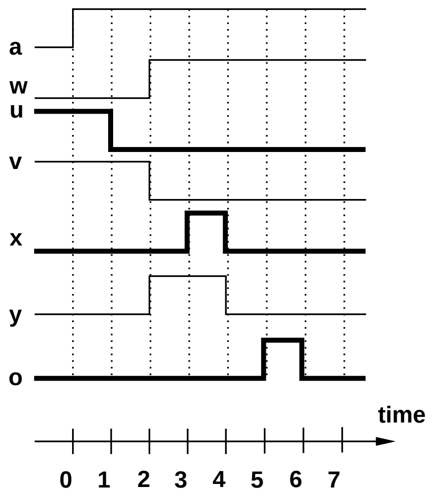
Example



(a)



(b)



(c)

- Propagation delays: 2 units.
- Shaded gate: 3 units and 1 unit.

Static co-sensitization

- Assumption:
  - Circuit modeled by AND, OR, INV gates.
  - INV are irrelevant to the analysis.
  - Floating mode.
- Controlling values:
  - 0 for AND gate.
  - 1 for OR gate.
- Gate has *controlled value*.

Static co-sensitization

- Path: $P = (v_{x_0}, v_{x_1}, \ldots, v_{x_m})$.

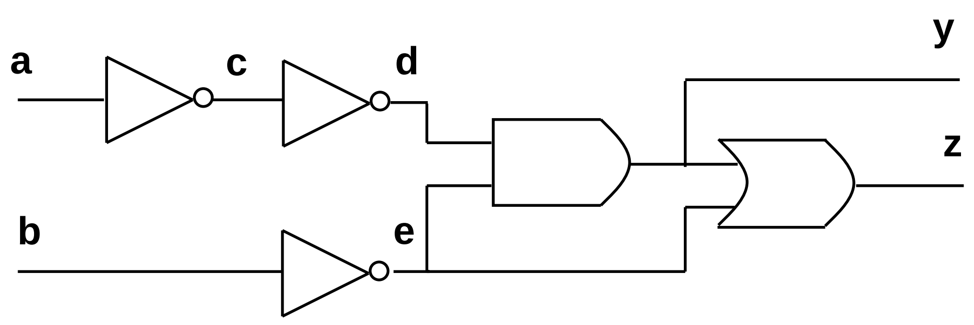
- A vector *statically co-sensitizes* a path to 1 (or to 0) if
  - $x_m = 1$ or (0) and
  - $v_{x_{i-1}}$ has a controlling value whenever v_{x_i} has a controlled value.

- Necessary condition for a path to be true.

False path detection test

- For all input vectors, one of the following is true:
  - (1) A gate is controlled and
    - the path provides a non-controlling value
    - a side-input provides a controlling value.
  - (2) A gate is controlled and
    - the path and a side-input have controlling values
    - the side-input presents the controlling value first.
  - (3) A gate is not controlled and
    - a side-input presents the non-controlling value last.

Example



- Path: $(v_a, v_c, v_d, v_y, v_z)$.
- For $a = 0, b = 0$
  - condition (1) occurs at the OR gate.
- For $a = 0, b = 1$
  - condition (2) occurs at the AND gate.
- For $a = 1, b = 0$
  - condition (2) occurs at the OR gate.
- For $a = 1, b = 1$
  - condition (1) occurs at the AND gate.

Important problems

- Check if circuit works at speed $\bar{t}$.
  - Verify that all true paths are faster than $\bar{t}$.
  - Show that all paths slower than $\bar{t}$ are false.
- Compute groups of false paths.
- Compute critical true path:
  - Binary search for values of $\bar{t}$.
  - Show that all paths slower than $\bar{t}$ are false.

Algorithms for delay minimization

- Alternate:
  - Critical path computation.
  - Logic transformation on critical vertices.

- Consider *quasi critical paths*:
  - Paths with near-critical delay.
  - Small slacks.

Algorithms for delay minimization

```
$REDUCE\_DELAY( G_n(V,E) , \epsilon)\{$
    **repeat** $\{$
        Compute critical paths and critical delay $\tau$;
        Set output required data-ready times to $\tau$;
        Compute slacks;
        $U$ = vertex subset with slack lower than $\epsilon$;
        $W$ = select vertices in $U$;
        Apply transformations to vertices $W$;
    $\}$**until** (no transformation can reduce $\tau$ );
$\}$
```

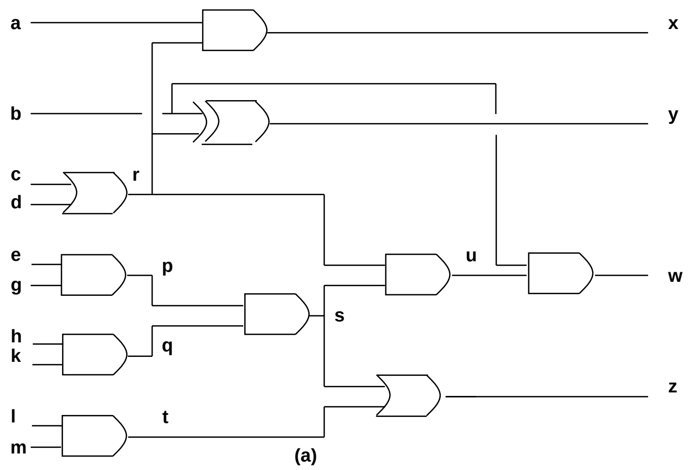
Transformations for delay reduction

- Reduce propagation delay.
- Reduce dependencies from critical inputs.
- *Favorable* transformation:
  - Reduces local data-ready time.
  - Any data-ready time increase at other vertices is bounded by the local slack.

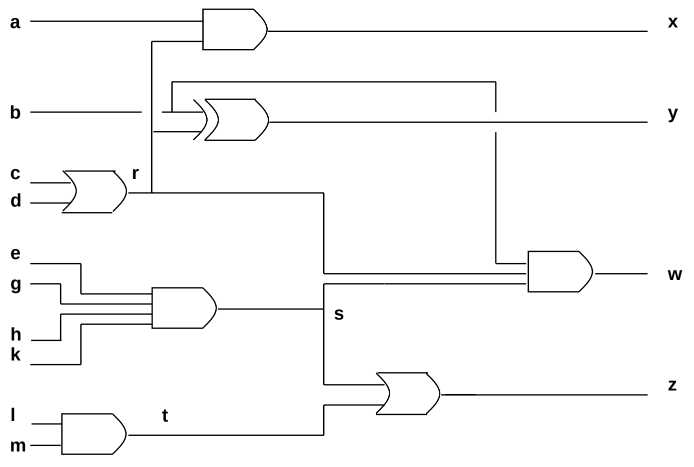
Example

- Unit gate delay.
- Transformation:
  - Elimination.
- Always favorable.
- Obtain several area/delay trade-off points.

Example



(a)



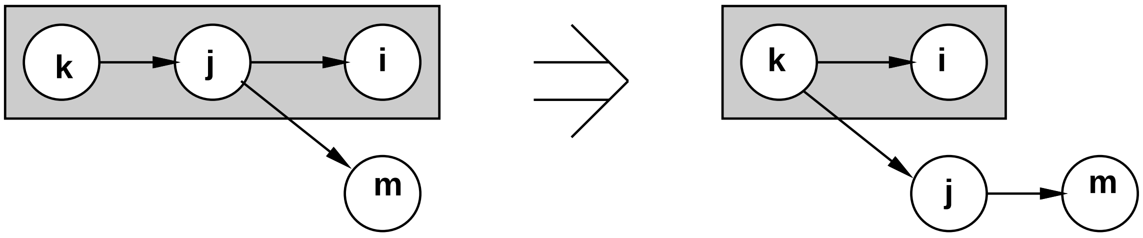
(b)

- Iteration 1: eliminate v_p, v_q. (No literal increase.)
- Iteration 2: eliminate v_u. (No literal increase.)
- Iteration 3: eliminate v_r, v_s, v_t. (Literals increase.)

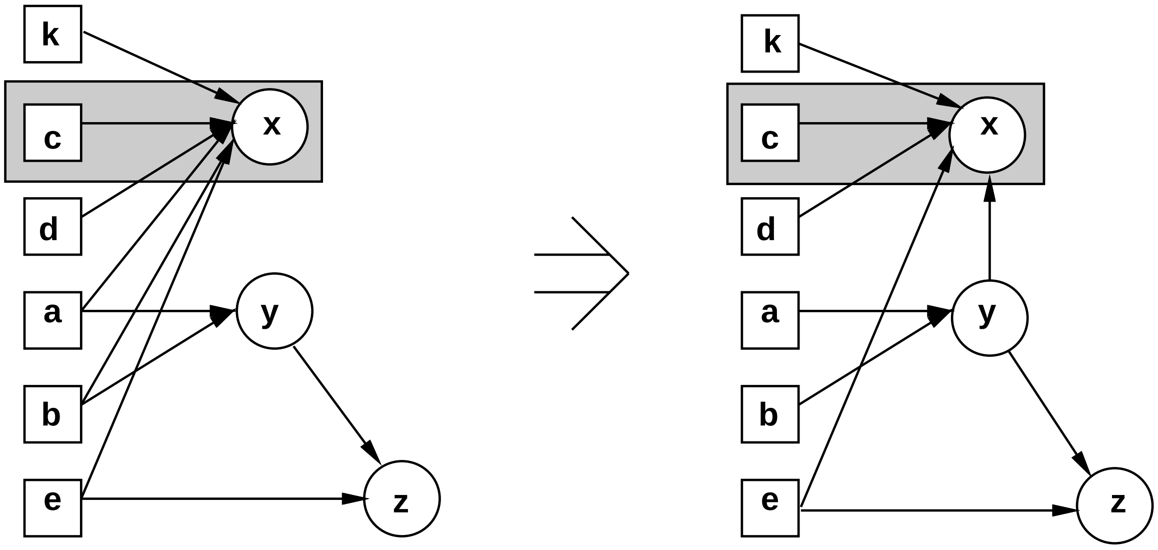
More refined delay models

- Elimination:
  - Reduces one stage.
  - Yields more complex and slower gates.
  - May slow other paths.
- Substitution:
  - Adds one dependency.
  - Loads and slows a gate.
  - May slow other paths.

Example

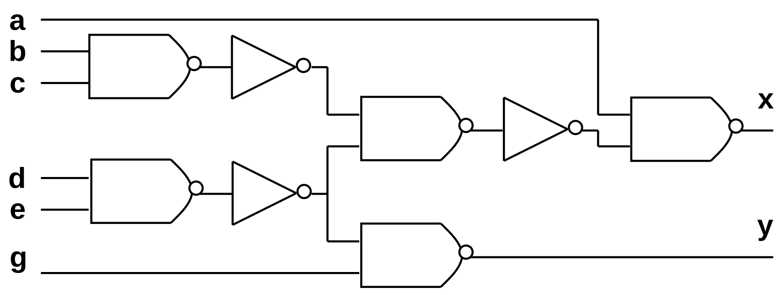


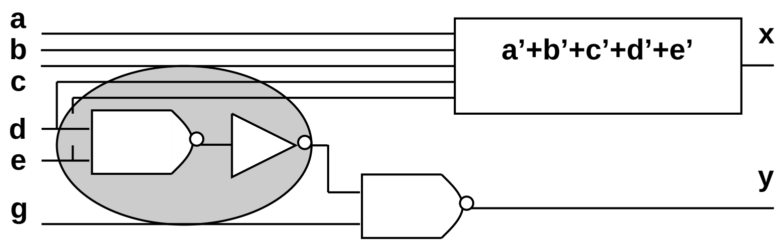
(a) (b)

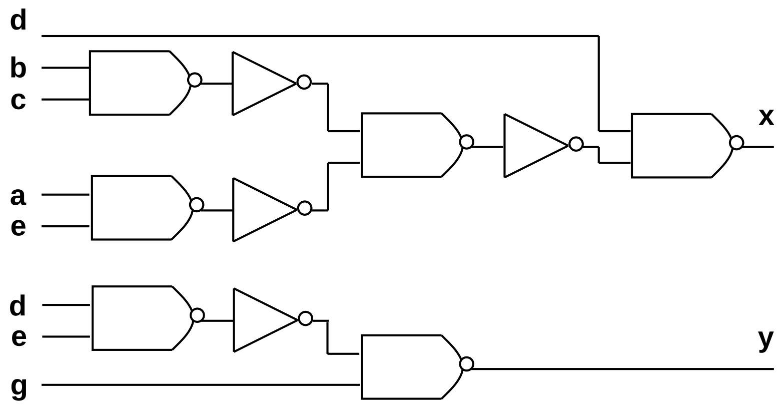


(c) (d)

Example







- NAND delay =2. INVERTER delay =1.
- All input data-ready are 0, except $t_d = 3$.

Speed-up algorithm

- Determine a subnetwork W of depth d.
- Collapse subnetwork by elimination.
- Duplicate vertices with successors outside W:
  - Record *area penalty*.
- Resynthesize W by timing-driven decomposition.
- Heuristics:
  - Choice of W.
  - Monitor *area penalty* and *potential speed-up*

Algorithms for minimal-area synthesis under delay constraints

- Make network *timing feasible*.
  - May not be possible.

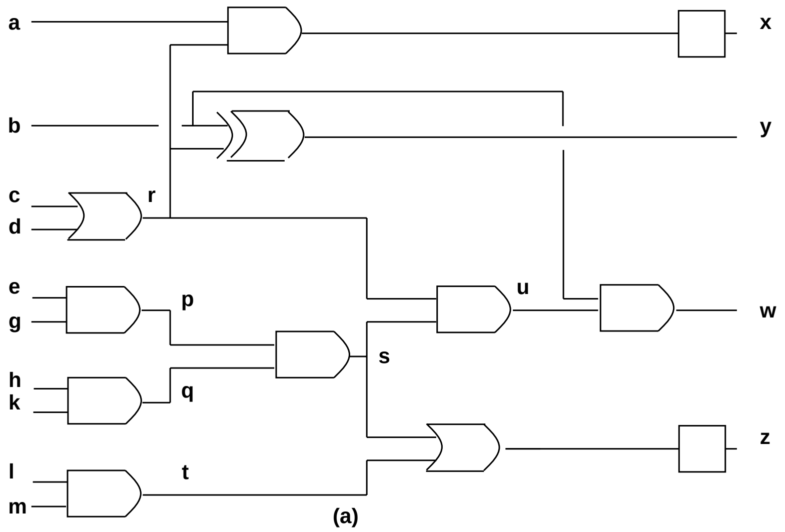
- *Minimize area* while preserving timing feasibility.
  - Use area optimization algorithms.
  - Monitor delays and slacks.
  - Reject transformations yielding negative slacks.

Making a network timing feasible.

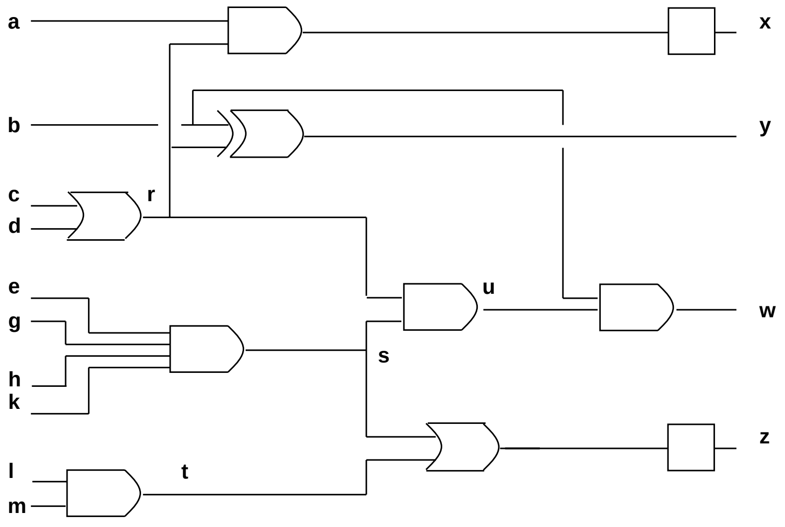
- Naive approach:
  - Mark vertices with negative slacks.
  - Apply transformations to marked vertices.
- Refined approach.
  - Transform multiple I/O delay constraints into single constraint by delay padding.
  - Apply algorithms for CP minimization.
  - Stop when constraints are satisfied.

Example

$$\bar{\mathbf{t}} = [2332]^T$$



(a)



(b)

Summary

- Timing optimization is crucial for achieving competitive logic design.

- Timing optimization problems are hard:
  - Detection of critical paths.
    - Elimination of false paths.
  - Network transformations.